Chapter-5

Image Enhancement and Restoration

QUESTION 1

Consider the image $\begin{pmatrix} 6 & 7 & 3 \\ 5 & 2 & 4 \\ 1 & 2 & 3 \end{pmatrix}$ $\begin{pmatrix} 6 & 7 & 3 \\ 5 & 2 & 4 \\ 1 & 2 & 3 \end{pmatrix}$. Perform the log, square root, and exponential functions and show the results.

The given image is:

Log function

6	7	3
5	2	4
1	2	3

Applying log base 10

0.778	0.845	0.477
0.699	0.301	0.602
0	0.301	0.477

Rounding off

1	1	0
1	0	1
0	0	0

Square Root function

6	7	3
5	2	4

Applying Square Root

2.44949	2.646	1.732
2.23607	1.414	2

Rounding off

2	3	2
2	1	2

1	2	3

1	1.414	1.732

1	1	2

Exponential function

Applying Square Root >>

6	7	3
5	2	4
1	2	3

403.429	1097	20.09
148.413	7.389	54.6
2.71828	7.389	20.09

Rounding off >>

403	1097	20
148	7	55
3	7	20

Apply threshold again on values greater than 255

255	255	20
148	7	55
3	7	20

QUESTION 2

What would be the impact of the removal of the last bit in the histogram for the following image? $\begin{pmatrix} 6 & 7 & 1 \\ 5 & 1 & 4 \\ 1 & 2 & 3 \end{pmatrix}$

Original image

6	7	1
5	1	4

Binary representation

110	111	001
101	001	100

1	2	3

001	010	011

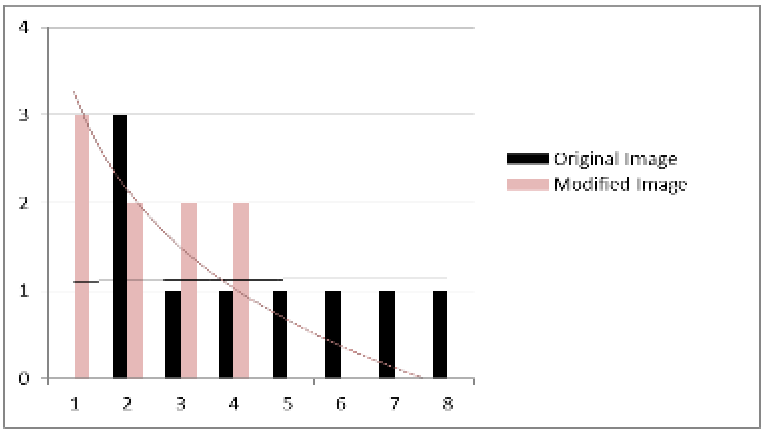
After removal of last bit

11	11	00
10	00	10
00	01	01

Decimal representation of the new image

3	3	0
2	0	2
0	1	1

Histogram comparison: Below is a comparison of the two histograms i.e. original and modified images. We notice the following differences in the histograms



a. The range of the histogram has decreased. Original range was 7, while the modified histogram has a range of 4.
b. Lower range means that fewer bits are sufficient to represent the image. In this case, the modified image can be represented with only 2 bits.

QUESTION 3

An image is in the range 10–50. To display the image on a device that has the grey-level range 0–255, what is the linear transformation that is required?

The transformation function T is given by the formula

$$T = (f(x,y) - 10)\frac{255 - 0}{50 - 10} + 0$$

or after simplifying,

$$T = (f(x,y) - 10)\frac{255}{40}$$

Question 4

1. Let the image f(x,y) be $\begin{pmatrix} 1 & 3 & 5 \\ 4 & 4 & 3 \\ 5 & 2 & 2 \end{pmatrix}$ and the reconstructed image be $\hat{f}(x,y) = \begin{pmatrix} 1 & 2 & 4 \\ 4 & 4 & 2 \\ 5 & 2 & 1 \end{pmatrix}$, What is MSE, SNR and PSNR for an 8-bit image. What would you do if the reference image is not given?

SOLUTION:

Solution:

$$MSE = \frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}[f(x,y) - \hat{f}(x,y)]^2$$

$$MSE = \frac{1}{3^2}[(1-1)^2 + (3-2)^2 + (5-4)^2 + (4-4)^2 + (4-4)^2 + (3-2)^2 + (5-5)^2 + (2-2)^2 + (2-1)^2]$$

$$= \frac{1}{9}[4] = 0.4444$$

$$SNR = 20\log_{10}(\frac{(0^2 + 1^2 + 1^2 + 0^2 + 0^2 + 1^2 + 0^2 + 0^2 + 1^2)}{9})$$

$$= 20\log_{10}(\frac{4}{9}) \cong -7.0437dB$$

$$PSNR = 20\log_{10}(\frac{255^2 \times 9}{4/9}) \cong 122.39\text{dB}$$

QUESTION 5

Consider the following image:

4	4	4	4	4
3	4	5	4	3
3	5	5	5	3
3	4	5	4	3

4	4	4	4	4

Write a procedure for histogram equalization and apply this to the image.

The procedure for histogram equalization involves calculations that are detailed as below:

Pixel Value	Number of occurrences	Cumulative occurrence	Normalized occurrence	Multiply by maximum gray value i.e. 7	Round off
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	6	6	0.24	1.68	2
4	14	20	0.8	5.6	6
5	5	25	1	7	7
6	0	25	1	7	7
7	0	25	1	7	7
Total		25	NA		

The above procedure gives us a new mapping. We apply this new mapping of intensities to pixel values to the original image to get a new image.

4	4	4	4	4
3	4	5	4	3

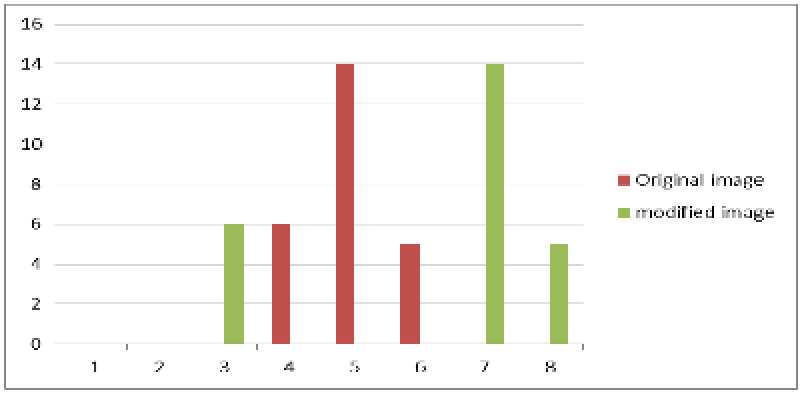
6	6	6	6	6
2	6	7	6	2

3	5	5	5	3
3	4	5	4	3
4	4	4	4	4

>

2	7	7	7	2
2	6	7	6	2
6	6	6	6	6

The changes to the histogram are shown below:



The new image has a histogram with a wider spread ie. Range of 5 versus a range of 3 on the original image's histogram.

QUESTION 6

Perform image enhancement for the 8 × 8 image distributions shown in the following tables..

a.

p_k	8	10	10	2	12	16	4	2
r_k	0	1	2	3	4	5	6	7

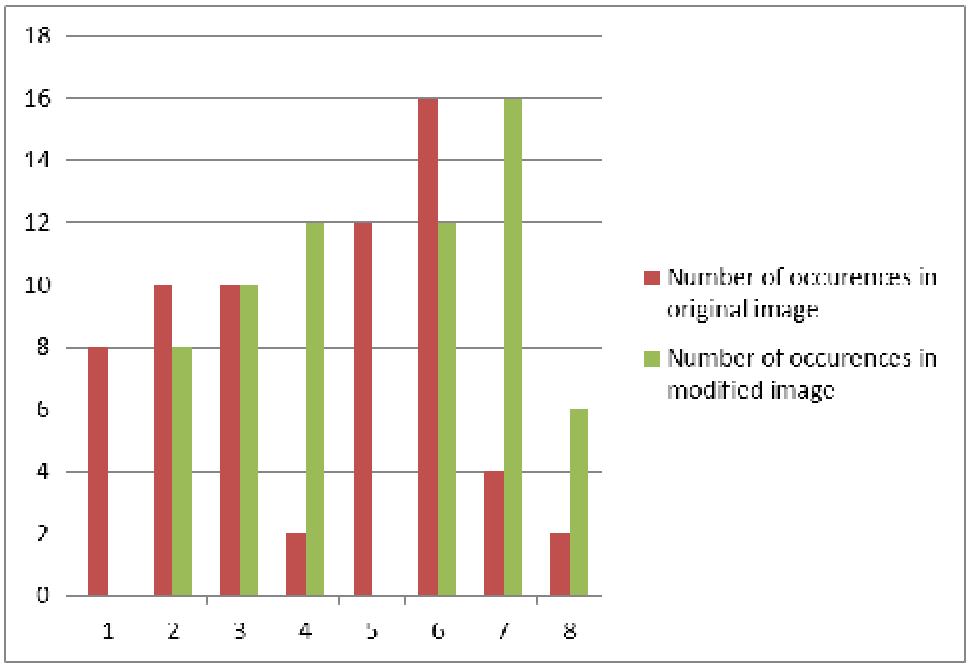
Histogram equalization is a means of image enhancement. The process of histogram equalization is shown below

Pixel Intensity Value	Number of occurences	Cumulative occurrence	Normalized occurrence	Multiply by maximum gray value i.e. 7	Round off
0	8	8	0.125	0.875	1
1	10	18	0.28125	1.96875	2
2	10	28	0.4375	3.0625	3
3	2	30	0.46875	3.28125	3
4	12	42	0.65625	4.59375	5
5	16	58	0.90625	6.34375	6
6	4	62	0.96875	6.78125	7
7	2	64	1	7	7
Total	64	NA			

We now have the modified mappings in the image.

Pixel intensity values	Number of occurences in original image	Number of occurences in modified image
0	8	0
1	10	8
2	10	10
3	2	12
4	12	0
5	16	12
6	4	16
7	2	6

The comparative histograms are shown below:



b.

p_k	2	2	10	10	20	8	6	8
r_k	0	1	2	3	4	5	6	7

We can repeat the same steps for this image mapping as well

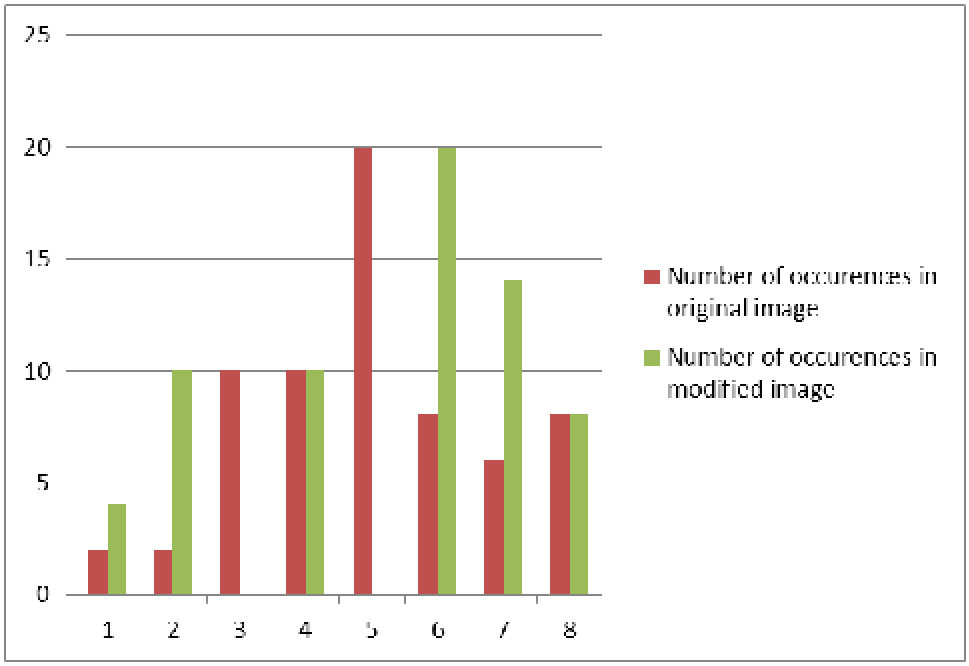
Pixel Intensity Value	Number of occurences	Cumulative occurrence	Normalized occurrence	Multiply by maximum gray value i.e. 7	Round off
0	2	2	0.030	0.212	0
1	2	4	0.061	0.424	0
2	10	14	0.212	1.485	1
3	10	24	0.364	2.545	3
4	20	44	0.667	4.667	5
5	8	52	0.788	5.515	6
6	6	58	0.879	6.152	6
7	8	66	1	7	7
Total		66	NA		

The modified mappings are:

Pixel intensity values	Number of occurences in original image	Number of occurences in modified image
0	2	4
1	2	10

2	10	0
3	10	10
4	20	0
5	8	20
6	6	14
7	8	8

And the comparative histograms are given below:



QUESTION 7

Perform histogram equalization n the following image.

$$\begin{pmatrix} 1 & 3 & 5 \\ 4 & 4 & 3 \\ 5 & 2 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 5 \\ 4 & 2 & 3 \\ 5 & 2 & 2 \end{pmatrix}$$

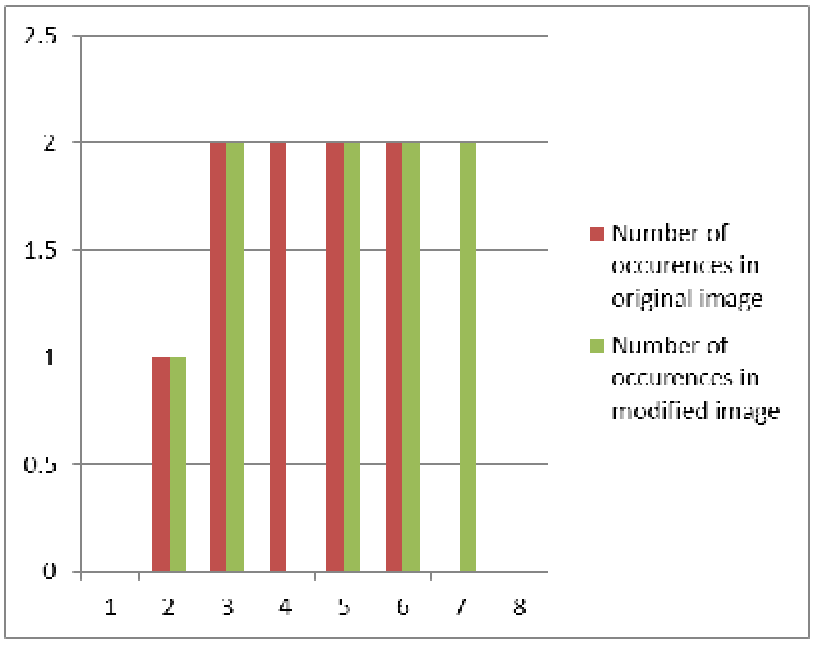
(a)

Pixel Intensity Value	Number of occurrences	Cumulative occurrence	Normalized occurrence	Multiply by maximum gray value i.e. 7	Round off
0	0	0	0.000	0.000	0
1	1	1	0.111	0.778	1
2	2	3	0.333	2.333	2
3	2	5	0.556	3.889	4
4	2	7	0.778	5.444	5
5	2	9	1.000	7.000	7
6	0	9	1.000	7.000	7
7	0	9	1	7	7
Total	9	NA			

Modified mappings

Pixel intensity values	Number of occurences in original image	Number of occurences in modified image
0	0	0
1	1	1
2	2	2
3	2	0
4	2	2
5	2	2
6	0	2
7	0	0

Comparative histograms



The modified image has a better histogram spread.

Problem (b) $\begin{pmatrix} 1 & 2 & 5 \\ 4 & 2 & 3 \\ 5 & 2 & 2 \end{pmatrix}$

Histogram equalization calculations:

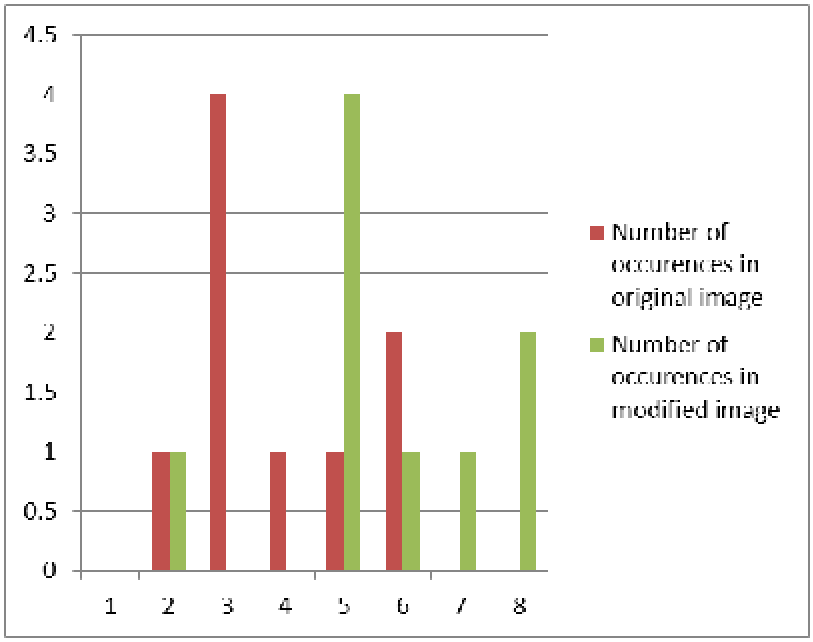
Pixel Intensity Value	Number of occurences	Cumulative occurrence	Normalized occurrence	Multiply by maximum gray value i.e. 7	Round off
0	0	0	0.000	0.000	0
1	1	1	0.111	0.778	1
2	4	5	0.556	3.889	4
3	1	6	0.667	4.667	5
4	1	7	0.778	5.444	5
5	2	9	1.000	7.000	7
6	0	9	1.000	7.000	7
7	0	9	1	7	7
Total	9	NA			

Modified mappings:

Pixel intensity values	Number of occurrences in original image	Number of occurrences in modified image
0	0	0
1	1	1

2	4	0
3	1	0
4	1	4
5	2	1
6	0	1
7	0	2

Comparative histogram



Modified image has a range of 6, while original has a range of 4.

QUESTION 8

Perform histogram specification to the problem 6 to the level shown in the following table.

P_k	0	0	0	0	20	20	16	8

We first calculate normalized distributions on the specified distribution requirements in a 8 x 8 image

Pixel intensity values	Desired Occurences	Cumulative # occurences	Normalized values * Max Gray value (7)	Round off
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	20	20	2.1875	2
5	20	40	4.375	4
6	16	56	6.125	6
7	8	64	7	7

Histogram specification Problem 6 (a)

Original intensity	Mapping after histogram equalization	Target	Values as per specification
0	0	0	4

1	1	0	4
2	2	0	4
3	4	0	5
4	5	2	6
5	7	4	7
6	7	6	7
7	7	7	7

Histogram specification Problem 6(b)

Original intensity	Mapping after histogram equalization	Target	Values as per specification
0	0	0	4
1	1	0	4
2	4	0	5
3	5	0	6
4	5	2	6
5	7	4	7
6	7	6	7
7	7	7	7

QUESTION 9

Apply the following filters on the given image and show the intermediate results.

a. Low-pass filter

b. High-pass filter

c. Median filter $\begin{pmatrix} 1 & 3 & 5 \\ 4 & 4 & 3 \\ 5 & 2 & 2 \end{pmatrix}$

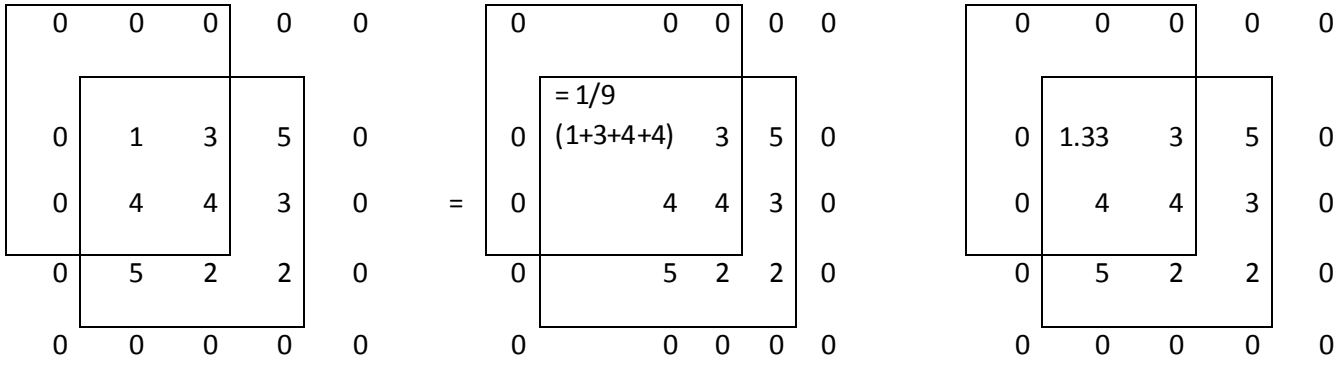
A simple Low pass filter is given by

$$\frac{1}{9}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Because this is a convolution, we first apply padding around the image. The easiest way to do this is to apply zero padding

0	*0*	*0*	*0*	*0*
0	1	3	5	*0*
0	4	4	3	*0*
0	5	2	2	*0*
0	*0*	*0*	*0*	*0*

Start by applying the filter to the upper left corner of the image



Move the mask around the entire image

1.33	2.22	1.67
2.11	3.22	2.11
1.67	2.22	1.22

Round off the fractions to get pixel values

1	2	2
2	3	2
2	2	1

Similarly, a high pass filter is obtained by using a Laplacian filter like:

$$\begin{pmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

As in the previous problem we can move around the mask on the zero padded image to get the following results

-3	2	14
6	4	1
14	-3	3

If we consider the above two filters without zero padding we get the following results

Low Pass Filter

Image				Change only center value				Round off		
1	3	5	=>	1	3	5	=>	1	3	5
4	4	3		4	3.2	3		4	3	3
5	2	2		5	2	2		5	2	2

High Pass Filter

Image				Change only centre value		
1	3	5	=>	1	3	5
4	4	3		4	4	3
5	2	2		5	2	2

Median Filter

List out the elements of the image in ascending order : 1,2,2,3,3,4,4,5,5

Replace the center of the image matrix with the median of the above data i.e. 3

Image

1	3	5
4	4	3
5	2	2

=>

Replace center with median

1	3	5
4	3	3
5	2	2

QUESTION 10

What is the result of applying the order-rank filter on the following image?

$$\begin{pmatrix} 1 & 3 & 5 \\ 4 & 4 & 3 \\ 5 & 2 & 2 \end{pmatrix}$$

First sort the elements in ascending order and list them to get the list : 1, 2, 2, 3, 3, 4, 4, 5, 5

The following are the effects of applying the order rank filters on the given image.

Median Filter

1	3	5

Min Filter

1	3	5

Max Filter

1	3	5

4	3	3
5	2	2

4	1	3
5	2	2

4	5	3
5	2	2

The median filter is used to suppress noise, while the min and max filters are used for gray level morphological transforms.