

Chapter-5 Image Enhancement

QUESTION 1

Consider the image $\begin{pmatrix} 6 & 7 & 3 \\ 5 & 2 & 4 \\ 1 & 2 & 3 \end{pmatrix}$. Perform the log, square root, and exponential functions and show the results.

The given image is:

Log function

Applying log base 10

6	7	3
5	2	4
1	2	3

Rounding off

0.778	0.845	0.477
0.699	0.301	0.602
0	0.301	0.477

1	1	0
1	0	1
0	0	0

Square Root function

Applying Square Root

6	7	3
5	2	4

Rounding off

2.44949	2.646	1.732
2.23607	1.414	2

2	3	2
2	1	2

1	2	3
---	---	---

1	1.414	1.732
---	-------	-------

1	1	2
---	---	---

Exponential function

Applying Square Root >>

Rounding off >>

6	7	3
5	2	4
1	2	3

403.429	1097	20.09
148.413	7.389	54.6
2.71828	7.389	20.09

403	1097	20
148	7	55
3	7	20

Apply threshold again on values greater than 255

255	255	20
148	7	55
3	7	20

QUESTION 2

What would be the impact of the removal of the last bit in the histogram for the following

image? $\begin{pmatrix} 6 & 7 & 1 \\ 5 & 1 & 4 \\ 1 & 2 & 3 \end{pmatrix}$

Original image

6	7	1
5	1	4

Binary representation

110	111	001
101	001	100

1	2	3
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001	010	011
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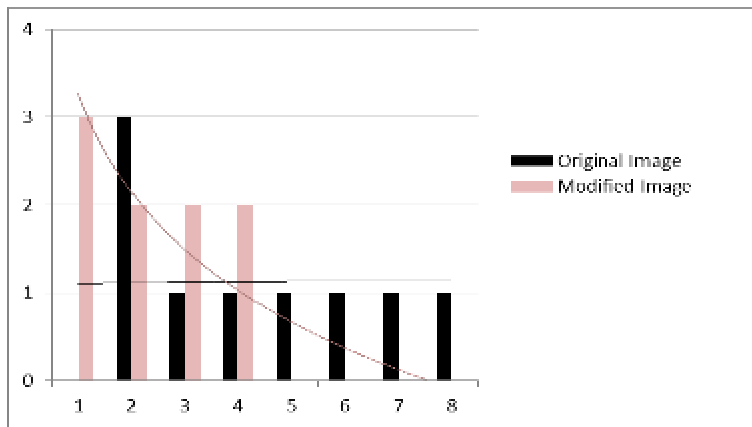
After removal of last bit

Decimal representation of the new image

11	11	00
10	00	10
00	01	01

3	3	0
2	0	2
0	1	1

Histogram comparison: Below is a comparison of the two histograms i.e. original and modified images. We notice the following differences in the histograms



- The range of the histogram has decreased. Original range was 7, while the modified histogram has a range of 4.
- Lower range means that fewer bits are sufficient to represent the image. In this case, the modified image can be represented with only 2 bits.

QUESTION 3

An image is in the range 10–50. To display the image on a device that has the grey-level range 0–255, what is the linear transformation that is required?

The transformation function T is given by the formula

$$T = (f(x, y) - 10) \frac{255 - 0}{50 - 10} + 0 \quad \text{or after simplifying,} \quad T = (f(x, y) - 10) \frac{255}{40}$$

Question 4

1. Let the image $f(x, y)$ be $\begin{pmatrix} 1 & 3 & 5 \\ 4 & 4 & 3 \\ 5 & 2 & 2 \end{pmatrix}$ and the reconstructed image be

$$\hat{f}(x, y) = \begin{pmatrix} 1 & 2 & 4 \\ 4 & 4 & 2 \\ 5 & 2 & 1 \end{pmatrix}, \text{ What is MSE, SNR and PSNR for an 8-bit image. What}$$

would you do if the reference image is not given?

SOLUTION:

Solution:

$$MSE = \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} [f(x, y) - \hat{f}(x, y)]^2$$

$$MSE = \frac{1}{3^2} [(1-1)^2 + (3-2)^2 + (5-4)^2 + (4-4)^2 + (4-4)^2 + (3-2)^2 + (5-5)^2 + (2-2)^2 + (2-1)^2]$$

$$= \frac{1}{9} [4] = 0.4444$$

$$SNR = 20 \log_{10} \left(\frac{(0^2 + 1^2 + 1^2 + 0^2 + 0^2 + 1^2 + 0^2 + 0^2 + 1^2)}{9} \right)$$

$$= 20 \log_{10} \left(\frac{4}{9} \right) \cong -7.0437 \text{ dB}$$

$$PSNR = 20 \log_{10} \left(\frac{255^2 \times 9}{4/9} \right) \cong 122.39 \text{ dB}$$

QUESTION 5

Consider the following image:

4	4	4	4	4
3	4	5	4	3
3	5	5	5	3
3	4	5	4	3

4	4	4	4	4
---	---	---	---	---

Write a procedure for histogram equalization and apply this to the image.

The procedure for histogram equalization involves calculations that are detailed as below:

Pixel Value	Number of occurrences	Cumulative occurrence	Normalized occurrence	Multiply by maximum gray value i.e. 7	Round off
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	6	6	0.24	1.68	2
4	14	20	0.8	5.6	6
5	5	25	1	7	7
6	0	25	1	7	7
7	0	25	1	7	7

Total 25 NA

The above procedure gives us a new mapping. We apply this new mapping of intensities to pixel values to the original image to get a new image.

4	4	4	4	4
3	4	5	4	3

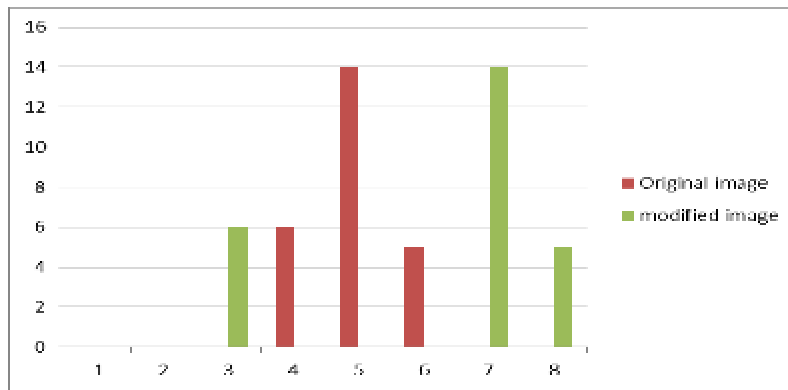
6	6	6	6	6
2	6	7	6	2

3	5	5	5	3
3	4	5	4	3
4	4	4	4	4

>

2	7	7	7	2
2	6	7	6	2
6	6	6	6	6

The changes to the histogram are shown below:



The new image has a histogram with a wider spread ie. Range of 5 versus a range of 3 on the original image's histogram.

QUESTION 6

Perform image enhancement for the 8×8 image distributions shown in the following tables..

a.

p_k	8	10	10	2	12	16	4	2
r_k	0	1	2	3	4	5	6	7

Histogram equalization is a means of image enhancement. The process of histogram equalization is shown below

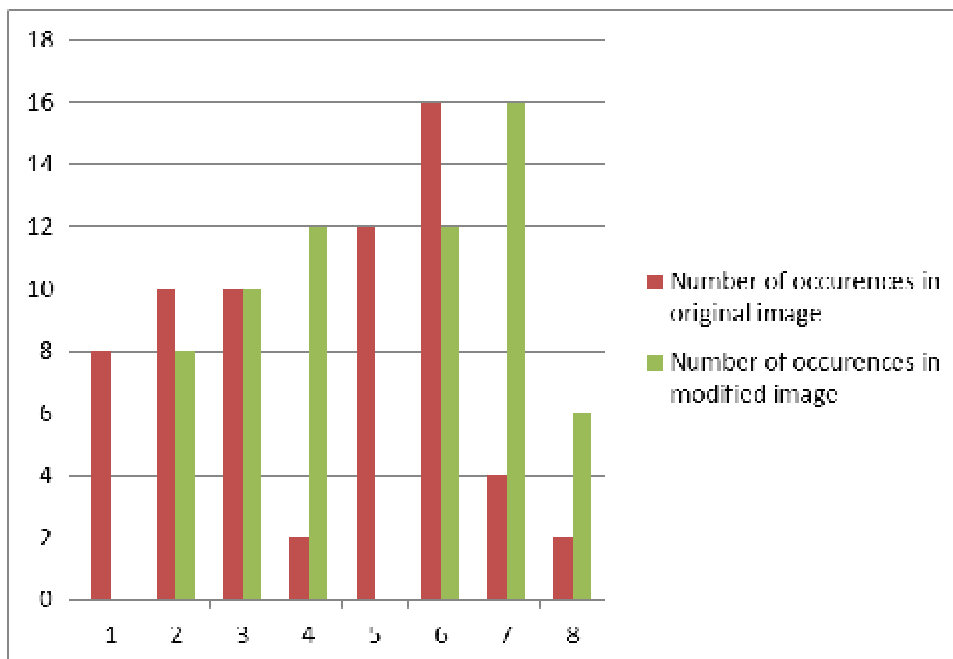
Pixel Intensity Value	Number of occurrences	Cumulative occurrence	Normalized occurrence	Multiply by maximum gray value i.e. 7	Round off
0	8	8	0.125	0.875	1
1	10	18	0.28125	1.96875	2
2	10	28	0.4375	3.0625	3
3	2	30	0.46875	3.28125	3
4	12	42	0.65625	4.59375	5
5	16	58	0.90625	6.34375	6
6	4	62	0.96875	6.78125	7
7	2	64	1	7	7

Total 64 NA

We now have the modified mappings in the image.

Pixel intensity values	Number of occurrences in original image	Number of occurrences in modified image
0	8	0
1	10	8
2	10	10
3	2	12
4	12	0
5	16	12
6	4	16
7	2	6

The comparative histograms are shown below:



b.

p_k	2	2	10	10	20	8	6	8
r_k	0	1	2	3	4	5	6	7

We can repeat the same steps for this image mapping as well

Pixel Intensity Value	Number of occurrences	Cumulative occurrence	Normalized occurrence	Multiply by maximum gray value i.e. 7	Round off
0	2	2	0.030	0.212	0
1	2	4	0.061	0.424	0
2	10	14	0.212	1.485	1
3	10	24	0.364	2.545	3
4	20	44	0.667	4.667	5
5	8	52	0.788	5.515	6
6	6	58	0.879	6.152	6
7	8	66	1	7	7

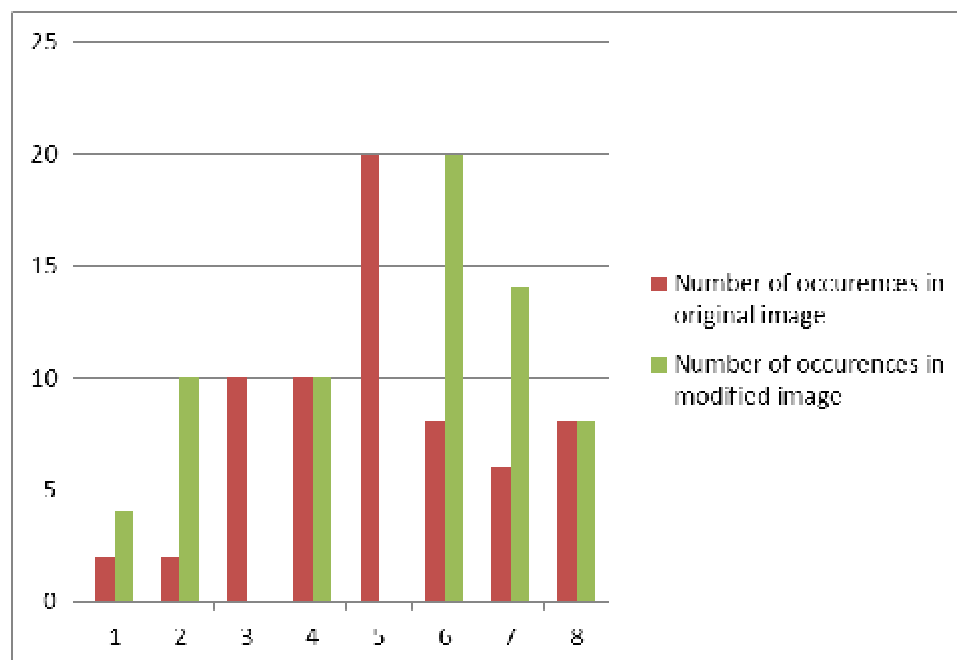
Total 66 NA

The modified mappings are:

Pixel intensity values	Number of occurrences in original image	Number of occurrences in modified image
0	2	4
1	2	10

2	10	0
3	10	10
4	20	0
5	8	20
6	6	14
7	8	8

And the comparative histograms are given below:



QUESTION 7

Perform histogram equalization on the following image.

$$\begin{pmatrix} 1 & 3 & 5 \\ 4 & 4 & 3 \\ 5 & 2 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 5 \\ 4 & 2 & 3 \\ 5 & 2 & 2 \end{pmatrix}$$

(a)

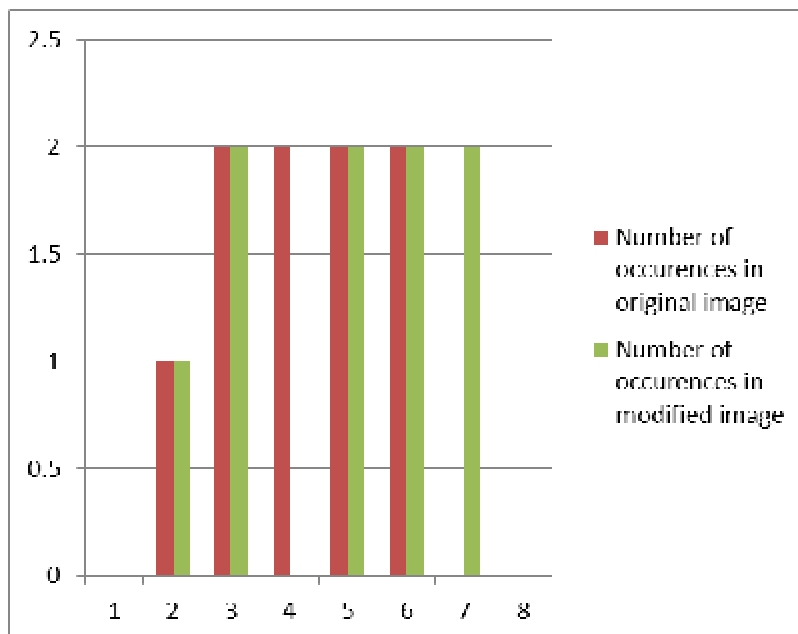
Pixel Intensity Value	Number of occurrences	Cumulative occurrence	Normalized occurrence	Multiply by maximum gray value i.e. 7	Round off
0	0	0	0.000	0.000	0
1	1	1	0.111	0.778	1
2	2	3	0.333	2.333	2
3	2	5	0.556	3.889	4
4	2	7	0.778	5.444	5
5	2	9	1.000	7.000	7
6	0	9	1.000	7.000	7
7	0	9	1	7	7

Total 9 NA

Modified mappings

Pixel intensity values	Number of occurrences in original image	Number of occurrences in modified image
0	0	0
1	1	1
2	2	2
3	2	0
4	2	2
5	2	2
6	0	2
7	0	0

Comparative histograms



The modified image has a better histogram spread.

Problem (b) $\begin{pmatrix} 1 & 2 & 5 \\ 4 & 2 & 3 \\ 5 & 2 & 2 \end{pmatrix}$

Histogram equalization calculations:

Pixel Intensity Value	Number of occurrences	Cumulative occurrence	Normalized occurrence	Multiply by maximum gray value i.e. 7	Round off
0	0	0	0.000	0.000	0
1	1	1	0.111	0.778	1
2	4	5	0.556	3.889	4
3	1	6	0.667	4.667	5
4	1	7	0.778	5.444	5
5	2	9	1.000	7.000	7
6	0	9	1.000	7.000	7
7	0	9	1	7	7

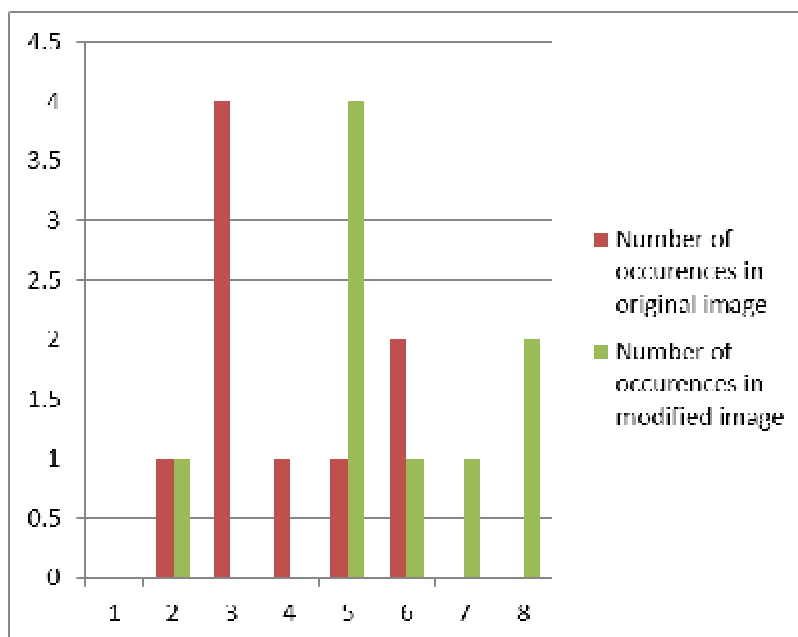
Total 9 NA

Modified mappings:

Pixel intensity values	Number of occurrences in original image	Number of occurrences in modified image
0	0	0
1	1	1

2	4	0
3	1	0
4	1	4
5	2	1
6	0	1
7	0	2

Comparative histogram



Modified image has a range of 6, while original has a range of 4.

QUESTION 8

Perform histogram specification to the problem 6 to the level shown in the following table.

P_k	0	0	0	0	20	20	16	8
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We first calculate normalized distributions on the specified distribution requirements in a 8 x 8 image

Pixel intensity values	Desired Occurences	Cumulative # occurences	Normalized values * Max Gray value (7)	Round off
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	20	20	2.1875	2
5	20	40	4.375	4
6	16	56	6.125	6
7	8	64	7	7

Histogram specification Problem 6 (a)

Original intensity	Mapping after histogram equalization	Target	Values as per specification
0	0	0	4

1	1	0	4
2	2	0	4
3	4	0	5
4	5	2	6
5	7	4	7
6	7	6	7
7	7	7	7

Histogram specification Problem 6(b)

Original intensity	Mapping after histogram equalization	Target	Values as per specification
0	0	0	4
1	1	0	4
2	4	0	5
3	5	0	6
4	5	2	6
5	7	4	7
6	7	6	7
7	7	7	7

QUESTION 9

Apply the following filters on the given image and show the intermediate results.

a. Low-pass filter

b. High-pass filter

c. Median filter $\begin{pmatrix} 1 & 3 & 5 \\ 4 & 4 & 3 \\ 5 & 2 & 2 \end{pmatrix}$

A simple Low pass filter is given by

$$\frac{1}{9} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Because this is a convolution, we first apply padding around the image. The easiest way to do this is to apply zero padding

0	0	0	0	0
0	1	3	5	0
0	4	4	3	0
0	5	2	2	0
0	0	0	0	0

Start by applying the filter to the upper left corner of the image

0	0	0
0	1	3
0	4	4
0	5	2
0	0	0

0	0	0
0	(1+3+4+4)	3
0	4	4
0	5	2
0	0	0

=

0	0	0
0	1.33	3
0	4	4
0	5	2
0	0	0

Move the mask around the entire image

1.33	2.22	1.67
2.11	3.22	2.11
1.67	2.22	1.22

Round off the fractions to get pixel values

1	2	2
2	3	2
2	2	1

Similarly, a high pass filter is obtained by using a Laplacian filter like:

$$\begin{pmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

As in the previous problem we can move around the mask on the zero padded image to get the following results

-3	2	14
6	4	1
14	-3	3

If we consider the above two filters without zero padding we get the following results

Low Pass Filter

			Change only							
Image			center value			Round off				
1	3	5	=>	1	3	5	=>	1	3	5
4	4	3		4	3.2	3		4	3	3
5	2	2		5	2	2		5	2	2

High Pass Filter

Image		Change only centre value																		
<table> <tr><td>1</td><td>3</td><td>5</td></tr> <tr><td>4</td><td>4</td><td>3</td></tr> <tr><td>5</td><td>2</td><td>2</td></tr> </table>	1	3	5	4	4	3	5	2	2	=>	<table> <tr><td>1</td><td>3</td><td>5</td></tr> <tr><td>4</td><td>4</td><td>3</td></tr> <tr><td>5</td><td>2</td><td>2</td></tr> </table>	1	3	5	4	4	3	5	2	2
1	3	5																		
4	4	3																		
5	2	2																		
1	3	5																		
4	4	3																		
5	2	2																		

Median Filter

List out the elements of the image in ascending order : 1,2,2,3,3,4,4,5,5

Replace the center of the image matrix with the median of the above data i.e. 3

Image		Replace center with median																		
<table border="1"> <tr><td>1</td><td>3</td><td>5</td></tr> <tr><td>4</td><td>4</td><td>3</td></tr> <tr><td>5</td><td>2</td><td>2</td></tr> </table>	1	3	5	4	4	3	5	2	2	=>	<table border="1"> <tr><td>1</td><td>3</td><td>5</td></tr> <tr><td>4</td><td>3</td><td>3</td></tr> <tr><td>5</td><td>2</td><td>2</td></tr> </table>	1	3	5	4	3	3	5	2	2
1	3	5																		
4	4	3																		
5	2	2																		
1	3	5																		
4	3	3																		
5	2	2																		

QUESTION 10

What is the result of applying the order-rank filter on the following image?

$$\begin{pmatrix} 1 & 3 & 5 \\ 4 & 4 & 3 \\ 5 & 2 & 2 \end{pmatrix}$$

First sort the elements in ascending order and list them to get the list : 1, 2, 2, 3, 3, 4, 4, 5, 5

The following are the effects of applying the order rank filters on the given image.

Median Filter	Min Filter	Max Filter									
<table border="1"> <tr><td>1</td><td>3</td><td>5</td></tr> </table>	1	3	5	<table border="1"> <tr><td>1</td><td>3</td><td>5</td></tr> </table>	1	3	5	<table border="1"> <tr><td>1</td><td>3</td><td>5</td></tr> </table>	1	3	5
1	3	5									
1	3	5									
1	3	5									

4	3	3
5	2	2

4	1	3
5	2	2

4	5	3
5	2	2

The median filter is used to suppress noise, while the min and max filters are used for gray level morphological transforms.