Chapter-13

Object Recognition

1. Consider two classes with the mean and variance given as follows:

(10 4) (8 3)

Use LDA and assign an instance *x* = 7 to one of the classes.

Solution:

$$d_1(x) = x^T m_1 - \frac{1}{2} m m^T$$
$$= (\mathrm{x}_1\ \mathrm{x}_2)^T (10\ 4) - \frac{1}{2}(10\ 4)(10\ 4)^T$$
$$= 10\mathrm{x}_1 + 4x_2 - 58$$

$$d_2(x) = x^T m_2 - \frac{1}{2} m_2 m_2^T$$
$$= (\mathrm{x}_1\ \mathrm{x}_2)^T (8\ 3) - \frac{1}{2}(8\ 3)(8\ 3)^T$$
$$= 8\mathrm{x}_1 + 3x_2 - 36.5$$

$$d_{12}(x) = d_1(x) - d_2(x)$$
$$= 2\mathrm{x}_1 + x_2 - 21.5$$

Substitute (7,7), into the equation. This gives the negative value. Therefore, the instance is assigned to the second class j.

2. Use Naïve Bayes classifier and classify the unknown pixel X. There are two types of class of pixels # and * is present in the image. The image is given below

$$\begin{pmatrix} \# & \# & * & * \\ \# & X & \# & * \\ * & \# & * & * \\ \# & \# & * & * \end{pmatrix}$$

Consider the 4-neighbour of X and determine the class of X.

Solution:

The prior probabilities are P(pixel= '#') = $\dfrac{\text{Number of \# pixels}}{\text{Total number of Pixels}} = \dfrac{7}{16} = 0.4$

P(Pixel = '*') = $\dfrac{\text{Number of * pixels}}{\text{Total number of Pixels}} = \dfrac{8}{16} = 0.5$

Given 4-neighbourhood of X, it is possible to calculate the likelihood of X.

Likelihood of X given '#' in 4-neighbourhood =

$$\frac{\text{Number of \# pixels in neighborhood of X}}{\text{Total number of '\#' Pixels}} = \frac{4}{7} = 0.57$$

Likelihood of X given '*' in 4-neighbourhood =

$$\frac{\text{Number of * pixels in neighborhood of X}}{\text{Total number of '*' Pixels}} = \frac{0}{8} = 0$$

Now posterior probability can be calculated as

[Prior probability of P(pixel= '#')] X [Likelihood of X given '#' in 4-neighbourhood]

= 0.4 X 0.57 = 0.228

[Prior probability of P(pixel= '*')] X [Likelihood of X given '*' in 4-neighbourhood]

= 0.5 X 0 = 0

Since 0.228 is greater than 0, The pixel X must be # only.

3. Consider the following data. Use naïve Bayesian classifier to classify the instance (1, 1).

S. no.	***X***	***Y***	**Class**
1	0	0	*C*1
2	1	0	*C*1
3	0	0	*C*2
4	1	1	*C*2

5	1	0	*C*1

Solution: Here $c_1 = 3$ and $c_2 = 1$

$P(c_2) = 1/4$; $P(c_1) = 3/4$ The conditional probability is estimated.

$P(a_{1=1}/c_1) = 2/3$; $P(a_2 = 1/c_2) = 0$

$P(a_2 = 1/c_1) = 0$; $P(a_2 = 1/c_2) = 1/1=1$

Therefore, $P(x/c_1) = P(a_1 = 1/c_1) * P(a_2 = 1/c_1)$

$= 2/3 * 0$

$= 0$

$P(x/c_2) = P(a_1 = 1/c_2) * P(a_2 = 1/c2)$

$= 0 * 1$

$= 0$

This is used to evaluate

$P(c_1/x) = 0.75 * 0$

$= 0$

$P(c_2/x) = 0.25 * 0$

$= 0$

Since $P(c_1/x) = P(c_2/x)$, the sample can belong to either c1 or c2.

4. Consider the following data:

(2 3 4) and (1 5 6)

(2 2 9) and (7 8 9)

Calculate the Euclidean and average Manhattan distance.

Solution:

(a) The Euclid distance is given as $\sqrt{(2-1)^2 + (3-5)^2 + (4-6)^2}$
$= \sqrt{1+4+4} = 3$

The average Manhattan distance is given as

$$\frac{1}{3}(|(2-1)| + |(3-5)| + |(4-6)|)$$
$$= \frac{1}{3}(5) = \frac{5}{3}$$

(b) For the points (2 2 9) and (7 8 9),

The Euclid distance is given as $\sqrt{(2-7)^2+(2-8)^2+(9-9)^2}$

$$= \sqrt{61}$$

The average Manhattan distance is given as

$$\frac{1}{3}(|(2-7)|+|(2-8)|+|(9-9)|)$$
$$=\frac{1}{3}(5+6)=\frac{11}{3}$$

5. Apply linear regression to the following data. Find Y values when X = 4.5 and X = 6.

S. no.	***X***	***Y***
1	3	5
2	7	8
3	12	5
4	16	9
5	20	8

Solution:

***Solution*:**

A linear regression model can be obtained as below. A line can be fit to the given data as $y = W_0 + W_1 x$.

$$\bar{x} = \frac{58}{5} = 11.6$$
$$\bar{y} = \frac{35}{5} = 7$$

$$W_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{(3-11.6)(5-7)+(7-11.6)(8-7)+(12-11.6)(5-7)+(16-11.6)(9-7)+(20-11.6}{[(3-11.6)^2+(7-11.6)^2+(12-11.6)^2+(16-11.6)^2+(20-11.6)^2]}$$
$$= \frac{29}{185.2} = 0.1565 \cong 0.157$$

$$W_0 = \bar{y} - W_1 \times \bar{x}$$
$$= 7\text{-}(0.157 \times 11.6)$$
$$= 5.1788$$

So the final equation is 5.1788 + 0.157 x

6. **Let us assume that Classifier performance with respect to a dataset is shown below. Evaluate the performance of the classifier.**

Expert vs classifier		Predicted class as per classifier	
		+	−
Actual class as per the expert	+	9 (TP)	0 (FN)
	−	1 FP	3 TN

Solution:

P = TP + FN = 9, N = FP + TN = 4

True Positive (TP-rate) = $\frac{TP}{P}$, where P = TP + FN = $\frac{9}{9} = 100\%$

False Positive (FP-rate) = $\frac{FP}{N}$, where N = FP + TN=$\frac{1}{4} = 0.25 = 25\%$

False Negative rate (FN-rate) = $\frac{FN}{P}$, where P = TP+FN = 0

True Negative rate (TN-rate) = $\frac{TN}{N}$, where P = TP+FN = $\frac{3}{9} = 0.33 = 33\%$

$$\text{Precision} = \frac{TP}{TP + FP} = \frac{9}{10} = 0.9 = 90\%\,;$$

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN} = \frac{12}{13} = 0.923 = 92.3\%$$

And error rate $= \dfrac{FP + FN}{TP + TN + FP + FN} = \dfrac{1}{13} \cong 0.0769 = 7.69\%$

7. Cluster the following data using hierarchical methods and show the dendrogram.

S. no.	***X***	***Y***
1	3	5
2	7	8
3	12	5
4	16	9
5	20	8

Solution:

The Euclid distance is given as

	1	2	3	4	5
1	-	5	9	13.6	17.26
2	5	-	5.83	9.05	13
3	9	5.83	-	5.66	8.54
4	13.6	9.05	5.66	-	4.1
5	17.26	13	8.54	4.1	-

The minimum is 4.1. Hence 4 & 5 should be clustered first. This results in the following table.

	1	2	3	{4,5}
1	-	5	9	13.6
2	5	-	5.83	9.05

3	9	5.83	-	5.66
{4,5}	13.6	9.05	5.66	-

The minimum is 5.0. Hence 1 & 2 should be clustered to yield the following table.

	{1,2}	3	4
{1,2}	-	5.83	-
3	5.83	-	5.66
{4,5}	-	5.66	-

This results in the cluster {(1, 2), (3), {4, 5}}

There is no purpose in progressing further as all the clusters will join together.

8. Let us assume five clusters with dominant elements spread as 3, 4, 5, 6, and 2. What is the purity of the cluster?

Solution:

There are 5 clusters – Therefore N = 5.

Purity = 1/5 * (3 + 4 + 5 + 6 + 2) = 20/5 = 4.

9. Consider the following data. Use *k*-means algorithm with *k* = 2 and show the result.

S. no.	***X***	***Y***
1	3	5
2	7	8
3	12	5
4	16	9

Solution:

Let k = 2. Let the initial seeds be (3,5) and (16,9).

The distance between (7,8) and (3,5) is 5

The distance between (7,8) and (16,9) is 9.05

So assign (7,8) to the cluster containing (3,5).

The new centroid is (3+7/2, 5 + 8/2) = (5,6.5)

For the sample (12,5),

The distance between (12,5) and (5,6.5) is 9.158

The distance between (12,5) and (16,9) is 5.6

So assign (12,5) to (16,9). The new centroid is (12+16/2, 5+9/2) = (14,7)

Check again the distance between (7,8) and (14,7), the distance is 7.07 but the distance between (7,8) and (5,6.5) is 4.27.

Hence the final clusters are { (3,5) (7,8)} and { (12,5) (16,9) }.

10. A 2 X 2 contingency table of a clustering algorithm is given as below. What is the value of Jaccard coefficient and Rand statistic.

	Same Cluster	Different Cluster
Same Class	5	3
Different Class	3	5

Solution:

Jacard Coefficient = $\frac{A}{B+C+D}=\frac{5}{11}=0.454$

Rand Statistic = $\frac{A+B}{A+B+C+D}=\frac{8}{16}=0.5$