Chapter-7

Image Compression

1. Calculate entropy for the following image:

Symbols	1	2	3	4	5	6
Rk	0.4	0.2	0.2	0.1	0.08	0.02

Solution:

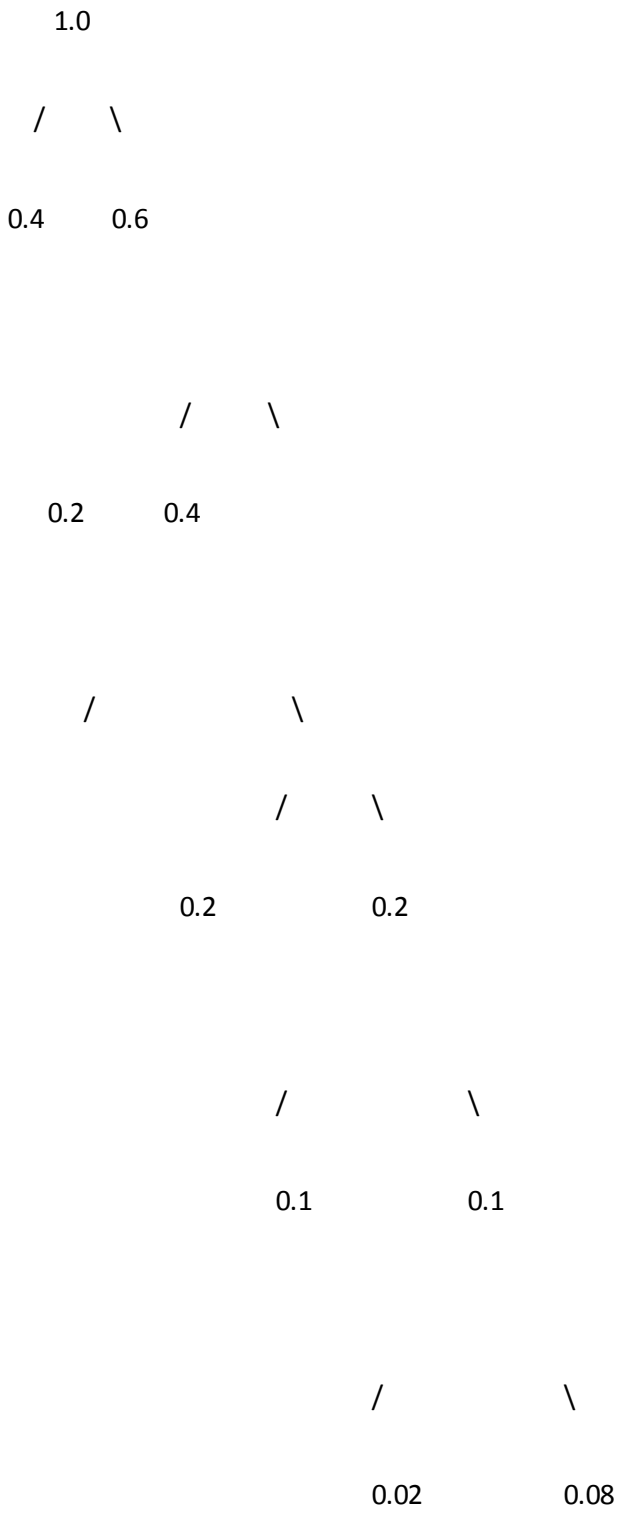
= 0.4 * log (0.4) + 0.2 * log (0.2) + 0.2 * log(0.2) + 0.1 log(0.01) + 0.08 log(0.08) + 0.02 * log (0.02)

= - [-0.5288-0.4644-0.4644-0.3322-0.2915-0.1129]

= 2.1942

2. Construct the Huffman code for the image given in Question 1

Solution:



The codes are

(1) : 0

(2) : 10

(3) : 110

(4) : 1110

(5): 11110

(6): 11111

3. Construct the RLC for the following images:

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

Solution :

1 1 1 2 2 2 1 2 2

(3,1), (2,3),(1,1), (2,2)

4. Construct the Shannon – Fano Tree for the above image

Solution: The frequency table is

1 2

6 3

So the corresponding tree is

o

/ \

(1) (2)

5. Demonstrate the predictive coding algorithm for the following pixels:

Symbols	**Character to be sent**
23	**23**
64	**64-23 = 41 (Store 31 – This takes 5 bits + 1 bit for sign for lossy Compression. Use original for lossless compression.**
39	**39-64=-25**

47	**47-39 = 8**
55	**55-47 = 8**
63	**63-55=8**

6. With the following symbols and their probabilities of occurrence, encode the message ABCD using the arithmetic coding algorithm.

Symbol	A	B	C	D
Probability	0.4	0.3	0.2	0.1

Solution:

First Character to be encoded is A. This falls in the range 0-0.4. So the range is 0.4-0.0 = 0.4. The new range is obtained as

A : 0.0 + 0.4 * 0.4 = 0.16

B : 0.16 + 0.3 * 0.4 = 0.28

C : 0.28 + 0.2 * 0.4 = 0.36

D : 0.36 + 0.1 * 0.4 = 0.4

The next Character to be encoded is B. This falls in the range 0.16-0.28. So the range is 0.28-0.16=0.12. The new range is obtained as

A : 0.16 + 0.4 * 0.12 = 0.208

B : 0.208 + 0.3 * 0.12 = 0.244

C : 0.244 + 0.2 * 0.12 = 0.268

D : 0.268 + 0.1 * 0.12 = 0.28

The next Character to be encoded is C. This falls in the range 0.244-0.268. So the range is 0.268-0.244=0.024. The new range is obtained as

A : 0.024 + 0.4 * 0.024= 0.2536

B : 0.2536 + 0.3 * 0.024 = 0.2608

C : 0.2608 + 0.2 * 0.024 = 0.2656

D : 0.2656 + 0.1 * 0.024 = 0.268

The last character to be sent is D.

So the arithmetic coding for the message "ABCD" is in the range between 0.2656 – 0.268

8. Code the following sentence using arithmetic coding algorithm:

SWISS

Solution:

The frequency is S: 3/5 = 0.6

W: 1/5 = 0.2

I : 1/5 = 0.2

The first character to be encoded is "S". So the range is 0.6-0 = 0.6. The new range is

S = 0 + 0.6 * 0.6 = 0.36

W = 0.36 + 0.2 * 0.6 = 0.48

I = 0.48 + 0.2 * 0.6 = 0.6

The next character to be encoded is "W". So the range is 0.36-0.48 = 0.12. The new range is

S = 0.36+0.6 * 0.12 = 0.432

W = 0.432 + 0.2 * 0.12 = 0.456

I = 0.456 + 0.2 * 0.12 = 0.48

The next character to be encoded is "I". So the range is 0.48-0.456=0.024. The new range is

S = 0.456+0.6 * 0.024 = 0.4704

W = 0.4704 + 0.2 * 0.024 = 0.4752

I = 0.4752 + 0.2 * 0.024 = 0.48

The next character to be encoded is "S". So the range is 0.4704-0.456=0.0144. The new range is

S = 0.456+0.6 * 0.0144 = 0.46464

W = 0.46464 + 0.2 * 0.0144 = 0.46752

I = 0.46752 + 0.2 * 0.0144 = 0.4704

The next character to be encoded is "S" of the message 'SWISS".

So the range is between 0.456-0.46464.

9. Assuming the quantization thresholds of 32, 48, and 64. Derive the quantization error for each of the following DCT coefficients:

127, 172, 167, 178. −164, and −128

Solution:

Value	Quantization Value	De-Quantization Value	Error
127	127/32=3.96 Approx. 4	4 X 32 = 128	-1
172	172/32=5.375 Approx. 5	5 X 32 = 160	12
167	167/32=5.21875 Approx. 5	5 X 32 = 160	7
178	178/32=5.6525 Approx. 6	6 X 32 = 192	-14
-164	-164/32=-5.125 Approx.-5	-5 X 32 = -160	-4
-128	-128/32=-4 Approx. -4	-4 X 32 = -128	0

Value	Quantization Value	De-Quantization Value	Error
127	127/48=2.645 Approx. 3	3 X 48 = 144	-17
172	172/48=3.583 Approx. 4	4 X 48 = 192	-20

167	167/48=3.479 Approx. 3	3 X 48 = 144	-23
178	178/48=3.7 Approx. 4	4 X 48 = 192	-14
-164	-164/32=-3.41 Approx.-3	-3 X 48 = -144	-20
-128	-128/32=-2.66 Approx. -3	-3 X 48 = -144	16

If the

Quantization value is changed to 64, The above yields

Value	Quantization Value	De-Quantization Value	Error

127	127/64=1.98 Approx. 2	2 X 64 = 128	-1
172	172/64=2.68 Approx. 3	3 X 64 = 192	-20
167	167/64=2.60 Approx. 3	3 X 64 = 192	-25
178	178/64=2.78 Approx. 3	3 X 64 = 192	-14
-164	-164/64=-2.56 Approx.-3	-3 X 64 = -192	28
-128	-128/64=-2	-2 X 64 = -128	0